

USING MATLAB AS A TEACHING AND LEARNING TOOL FOR BEAM BENDING PROBLEMS IN MECHANICS

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This paper presents a MATLAB solution for the shear force, bending moment, and deflection as continuous functions of the distance measured from the left hand support for a simply supported beam carrying concentrated and uniformly distributed loads. The solution has been cast in such a way that it can easily be modified for the beam to carry any number of these loads. The solution provides an easy way of determining the maximum values of the functions and their locations. Using numerical values for a particular loading, the solution and graphs for the above functions are presented as obtained from an actual MATLAB script, which has been included at the end of the paper in Appendix 1. The reader who has access to MATLAB is encouraged to use the script to verify the results presented. Also mechanics lecturers who are involved in engineering undergraduate education will find the script very useful in terms of demonstrating the effect of various concentrated and uniform loads on simply supported beams.

Key words: standard beams, MATLAB scripts, numerical solutions, engineering education.

1 INTRODUCTION

Engineering education has metamorphosed over the last three decades, from the slide-rule based, hand-computed solutions to complex problems to the current software-based, graphically appealing solutions. In this paper we present a simple software solution to a typical undergraduate engineering problem: beam bending of a simply supported beam subjected to a combination of concentrated and uniformly distributed loads.

Standard solutions of the problem are well known and can be found in almost any textbook in solid mechanics or strength of materials [1,2]. For a beam carrying several of these loads, the formulation of the total solution can easily be done through the principle of superposition. However, carrying out the numerical solutions for such cases manually would be an impossible task. Also, for a general loading, determining the maximum values of the above-mentioned functions and their locations on the beam for design purposes is not an easy matter. In this paper the authors have demonstrated that using MATLAB software, the total solution for numerical values for any combination of the above-mentioned loading can very easily be achieved through careful programming. The utility of the software-based approach is demonstrated through the attached MATLAB script (Appendix 1), which any mechanics lecturer or student can run and visualize the effect of changing loading, whether its position, nature or quantum.

2 ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH TYPICAL LOADING

Consider a simply supported beam AB of length L supported at A by a pinned support and at B by a

roller support as shown in Fig.1 carrying the following given loads:

- (i) Its own self-weight, w_s .
- (ii) A uniformly distributed applied load, w_a , on the entire length of the beam.
- (iii) A point load P_1 at a known distance a_1 from the left hand support (LHS).
- (iv) A load P_2 at a known distance a_2 from LHS.
- (v) A uniformly distributed load u_1 on a known length s_1 starting at a known distance b_1 from LHS.
- (vi) A couple M_{a1} applied clockwise at a known distance c_1 from LHS.

We are interested in developing a MATLAB solution for the above beam that will determine the following:

- (i) The reactions R_A and R_B at A and B respectively.
- (ii) The shear force, $Q(x)$, as a function of x (where x is the distance from the LHS) and plot the shear force diagram (SFD) along the beam. The maximum shear force, Q_{max} , and its location. Clockwise shear will be taken as positive.
- (iii) The bending moment, $M(x)$, as a function of x and plot the bending moment diagram (BMD) along the beam. The maximum bending moment, M_{max} , and its location and hence the maximum bending stress and its location. Sagging bending moment will be taken as positive.
- (iv) The deflection function $y(x)$ along the beam and hence the maximum deflection, y_{max} , and its location. Deflection is measured positive downwards.

To simplify the MATLAB solution, it is best to consider the above problem as five separate problems and then use the principle of superposition at the end to get the total solution. This approach will also make the solution to be easily modified to take any number of the applied loads.

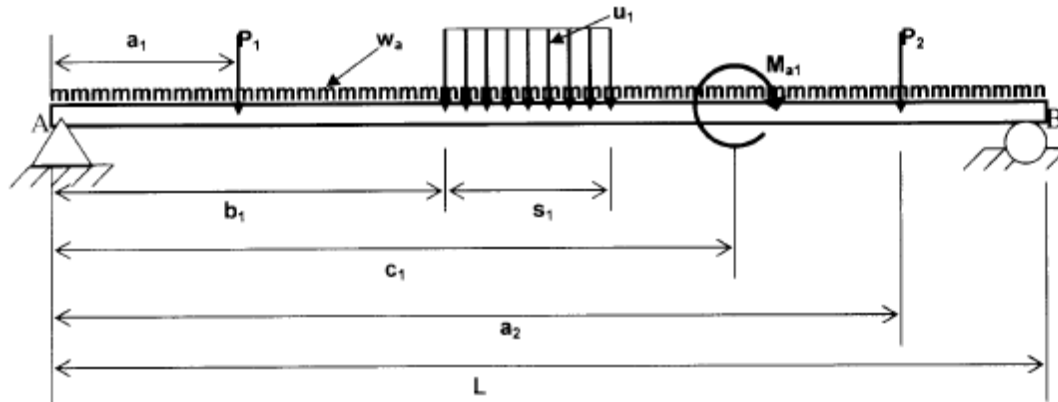


Fig. 1

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2.1 Problem 1: Self weight, w_s , and Applied Load, w_a , Acting Alone.

For this loading the shear force, $Q_1(x)$, and the bending moment, $M_1(x)$, are respectively given by $Q_1(x) = R_{A1} - wx$; (i) and $M_1(x) = R_{A1}x - wx^2/2$ (ii) where $w = w_s + w_a$ and R_{A1} is the reaction at A, yet to be determined. In this case it is clear that the reaction at A is equal to that at B and therefore,

$$R_{A1} = R_{B1} = wL/2 \tag{2}$$

Using the well known second order differential equation for the deflection and the double integration method [1, 2], the deflection, $y_1(x)$, at any value of x can be shown to be given by

$$y_1(x) = \frac{1}{EI} \left\{ -\frac{R_{A1}x^3}{6} + \frac{wx^4}{24} + K_{11}x \right\} \tag{3}$$

where K_{11} is a constant given by

$$K_{11} = R_{A1}L^2/6 - wL^3/24 \tag{4}$$

and where E is the Young's modulus of the beam material and I is the 2nd moment of area of the cross-section of the beam about the neutral axis of bending. For standard beams the value of I is obtained directly from the relevant standard. For non-standard beams I has to be computed from the geometry of the section. For the numerical solutions in this paper the standard beam used is I-PE-300, which is taken from the German Standard I-Beam DIN1025.

2.2 Problem 2: Point load P_1 Acting Alone

For this loading the shear force, $Q_2(x)$, and the bending moment, $M_2(x)$, are respectively given by

$$Q_2(x) = R_{A2} - P_1[x - a_1]^0; \text{ (i) and } \tag{5}$$

$$M_2(x) = R_{A2}x - P_1[x - a_1] \text{ (ii)}$$

where R_{A2} is the reaction at A, yet to be determined, and the square brackets are the usual Macaulay's method brackets signifying that the term is neglected when the value inside the bracket is negative. For the Macaulay's method, the reader is referred to Reference [1].

The reactions at A and B are respectively given by

$$R_{A2} = (L - a_1)P_1/L \text{ (i) and } \tag{6}$$

$$R_{B2} = a_1P_1/L \text{ (ii)}$$

and the deflection, $y_2(x)$, by

$$y_2(x) = \frac{1}{EI} \left\{ -\frac{R_{A2}x^3}{6} + \frac{P_1}{6}[x-a_1]^3 + K_{21}x \right\} \quad (7)$$

where K_{21} is a constant given by

$$K_{21} = \frac{R_{A2}L^2}{6} - \frac{P_1}{6L}(L-a_1)^3 \quad (8)$$

2.3 Problem 3: Point load P_2 Acting Alone

For this case, the shear force, $Q_3(x)$, and the bending moment $M_3(x)$ are respectively given by

$$\begin{aligned} Q_3(x) &= R_{A3} - P_2[x-a_2]^0; \text{ (i) and} \\ M_3(x) &= R_{A3}x - P_2[x-a_2] \text{ (ii)} \end{aligned} \quad (9)$$

and the reactions at A and B respectively by

$$\begin{aligned} R_{A3} &= (L-a_2)P_2/L \text{ (i) and} \\ R_{B3} &= a_2P_2/L \text{ (ii)} \end{aligned} \quad (10)$$

The deflection, $y_3(x)$, is given by

$$y_3(x) = \frac{1}{EI} \left\{ -\frac{R_{A3}x^3}{6} + \frac{P_2}{6}[x-a_2]^3 + K_{31}x \right\} \quad (11)$$

where K_{31} is a constant given by

$$K_{31} = \frac{R_{A3}L^2}{6} - \frac{P_2}{6L}(L-a_2)^3 \quad (12)$$

Clearly any additional number of point loads on the beam can be included in the analysis in the same way.

2.4 Problem 4: Uniformly Distributed Load u_1 Acting Alone

With reference to Fig. 1, the uniform load u_1 is extended to the right hand support (RHS). To nullify the effect of extending u_1 on top to the RHS, a uniform load u_1 is then placed on the bottom acting upwards starting at a distance $x=b_1+s_1$ from the LHS and extending all the way to the RHS [1]. The shear force $Q_4(x)$ can be shown to be given by

$$\begin{aligned} Q_4(x) &= R_{A4} - u_1[x-b_1] + \\ &u_1[x-(b_1+s_1)] \end{aligned} \quad (13)$$

where R_{A4} is the reaction at A, yet to be determined.

The bending moment, $M_4(x)$, is given by

$$\begin{aligned} M_4(x) &= R_{A4}x - \frac{u_1}{2}[x-b_1]^2 + \\ &\frac{u_1}{2}[x-(b_1+s_1)]^2 \end{aligned} \quad (14)$$

and the reactions at A and B respectively by

$$\begin{aligned} R_{A4} &= \frac{u_1}{2L} \left\{ (L-b_1)^2 - (L-b_1-s_1)^2 \right\} \text{ (i)} \\ \text{and } R_{B4} &= u_1s_1 - R_{A4} \end{aligned} \quad (15)$$

The deflection, $y_4(x)$, is given by

$$y_1(x) = \frac{1}{EI} \left\{ -\frac{R_{A1}x^3}{6} + \frac{u_1}{24}[x-b_1]^4 - \frac{u_1}{24}[x-(b_1+s_1)]^4 + K_{A1}x \right\} \quad (16)$$

where K_{A1} is a constant given by

$$K_{A1} = \frac{R_{A1}L^2}{6} + \frac{u_1}{24L} \left\{ (L-b_1-s_1)^4 - (L-b_1)^4 \right\} \quad (17)$$

It is also clear that any additional number of uniform loads acting on other parts of the beam can be included in the analysis in the same way.

2.5 Problem 5: Applied couple M_{A1} Acting Alone

For this case it is clear that the reactions are

$$\begin{aligned} R_{A5} &= -M_{A1}/L; \text{ (i)} \\ \text{and } R_{B5} &= M_{A1}/L \text{ (ii)} \end{aligned} \quad (18)$$

and shear force $Q_5(x)$ is a constant and given by

$$Q_5(x) = R_{A5} \quad (19)$$

The bending moment, $M_5(x)$, can be shown to be given by

$$M_5(x) = R_{A5}x + M_{A1}[x-c_1]^0 \quad (20)$$

and the deflection, $y_5(x)$, by

$$y_5(x) = \frac{1}{EI} \left\{ -\frac{R_{A5}x^3}{6} + \frac{M_{A1}}{2}[x-c_1]^2 + K_{51}x \right\} \quad (21)$$

where K_{51} a constant given by

$$K_{51} = \frac{R_{A5}L^2}{6} - \frac{M_{A1}}{2L}(L-c_1)^2 \quad (22)$$

Clearly any number of couples acting elsewhere on the beam can be included in the analysis in the same way.

2.6 Total Solution by the Principle of Superposition

The required total reaction R_A is given by

$$R_A = R_{A1} + R_{A2} + R_{A3} + R_{A4} + R_{A5} \quad (23)$$

and the required total reaction at B is given by

$$R_B = R_{B1} + R_{B2} + R_{B3} + R_{B4} + R_{B5} \quad (24)$$

The total shear force function is given by

$$Q(x) = Q_1(x) + Q_2(x) + Q_3(x) + Q_4(x) + Q_5(x) \quad (25)$$

The total bending moment function is given by

$$M(x) = M_1(x) + M_2(x) + M_3(x) + M_4(x) + M_5(x) \quad (26)$$

And the total deflection function is given by

$$y(x) = y_1(x) + y_2(x) + y_3(x) + y_4(x) + y_5(x) \quad (27)$$

The maximum values of the solution functions in equations (25), (26), and (27) can be determined in a

MATLAB solution and their locations on the beam can also be obtained from the same solution as explained below. The maximum bending stress, σ_{max} , is obtained from

$$\sigma_{max} = \frac{M_{max}}{Z} \quad (28)$$

where M_{max} is the maximum bending moment and Z is the section modulus of bending for the cross-section of the beam. For standard beams the value of Z is obtained directly from tables of the relevant standard.

3 MATLAB SOLUTION

In a MATLAB solution the continuous functions for the shear force, bending moment and deflection as given in equations (25), (26), and (27) above will be computed at discrete points on the beam starting from the LHS. The values computed at the discrete points will give the exact values of the functions at these points. However, the smoothness of the curves to be plotted for these functions when they are non-linear will clearly depend on how close the discrete points are made to each other. It is clear that very smooth curves for the solutions for these functions will be obtained by discretizing the beam into a very large number, n , of segments. However, if n is very large, the number of computations to be done by the computer on all the equations, some of them very complicated, for the total solution will also be very large and the computation time can be quite long. This should not be a problem on modern super fast computers.

For the numerical solutions presented in this paper, the values of the above functions have been computed at a spacing of 1 mm starting from the LHS. The variable x representing a varying beam length is created as a vector space $x(i)$ with n elements. The LHS, where $x=0$, is represented by the first element of the vector, i.e. $x(1)$ and the RHS, where $x=L$, by the n^{th} element $x(n)$. This means that the length L has been divided into $n-1$ intervals. If we want to compute the functions at a spacing of 1 mm, we use $n=1000L+1$, where L is in metres, and for a 2 mm spacing we use $n=500L+1$, and so on. In MATLAB the vector x is automatically created and stored in the workspace by the general `linspace` command, $x = \text{linspace}(0, L, n)$ and for the 1 mm spacing the command $x = \text{linspace}(0, L, (L*1000+1))$ was used.

Programming the above solution in MATLAB is straightforward for the reader familiar with it. However, great care must be taken to deal with the terms with the Macaulay's brackets. Programmes in MATLAB are written in M-files called script files. The script in Appendix 1 has been written with sufficient comment statements (the ones starting with

the % sign) to make it self-explanatory. The reader not familiar with MATLAB and programming in general is referred to References [3] and [4]. In a MATLAB solution, the values of the functions computed at the discrete points are stored as vectors and MATLAB provides a **max command** to operate on the vector variable to determine its maximum value. Once the maximum value is obtained, a simple **if loop** inside a **for loop** can be used to locate its position on the beam (See details of this in Appendix 1).

Plotting the shear force and bending moment diagrams, and the deflection curve can easily and quickly be done in MATLAB.

3.1 Several Applications of the MATLAB Script

The MATLAB script in Appendix 1 can be used to study the solutions for the following types of loading on a simply supported beam without having to alter the programme as presented:

- Uniform load over the entire length. In the data input part at the beginning of the programme, set $P1=P2=u1=Ma1=0.0$. Compare the output from the programme for M_{max} and y_{max} , which occur at mid-span, to the values obtained by the well-known formulae

$$M_{max} = wL^2 / 8; \quad \text{and}$$

$$y_{max} = 5wL^4 / 384$$

- One point load anywhere on a light beam: Set $u1=P2=wa=ws=Ma1=0.0$. A good example to demonstrate the effect a point load has on the SFD and BMD at its location.
- Two point loads anywhere on a light beam: Set $u1=wa=ws=Ma1=0.0$. Again a good example to demonstrate the effect of point loads on the SFD and BMD.
- One applied couple anywhere on a light beam: Set $u1=P1=P2=wa=ws=0.0$. Good example to demonstrate that the applied moment has no effect on the SFD at its point of application.
- Uniform load and one point load, P , at midspan: Set $P1=P2=P/2$, $a1=L/2$ and $a2=L-a1$; $u1=Ma1=0.0$. Compare the output for M_{max} and y_{max} , which occur at mid-span, to the values obtained from

$$M_{max} = wL^2 / 8 + PL / 4; \quad \text{and}$$

$$y_{max} = 5wL^4 / 384 + PL^3 / 48$$

3.2 MATLAB Results

The script in Appendix 1 was run with the following data for the beam I PE 300 from the German

Standard I-beam DIN-1025, of length $L=4$ m: its own self weight, $w_s=10$ kN/m, $P_1=100$ kN at $a_1=1.0$ m, $P_2=50$ kN at $a_2=2.0$ m, $u_1=15$ kN/m for $s_1=2.0$ m starting at $b_1=2.0$ m, and $M_{a1}=50$ kN.m at $c_1=3.0$ m.

The SFD is shown in Fig.2 from which the step effect of the point loads on the diagram at their points of application is clearly seen. Q_{max} has been computed as equal to 115.83 kN at $x=0.0$ and as expected it is equal to the reaction at A. The BMD is shown in Fig.3 from which the step effect of the applied moment on the diagram at its point of application is clearly seen. M_{max} has been computed as equal 112.03 kNm at a distance $x=1520$ mm. The maximum bending stress has been computed as equal to 201.13 MPa. The deflection curve is shown in Fig.4. The maximum deflection y_{max} has been computed as equal 9.47 mm at a distance $x=1779$ mm.

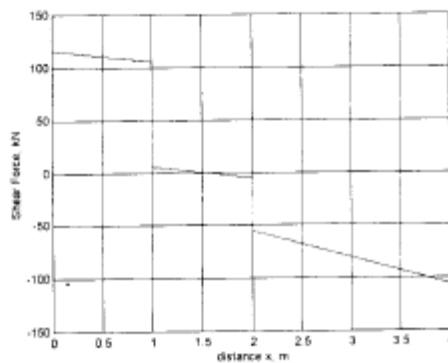


Fig. 2: SFD: $Q_{max}=R_A=115.83$ kN at LHS

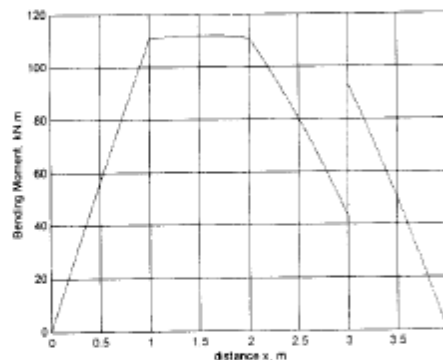


Fig.3: BMD: $M_{max} = 112.03$ kN.m at $x = 1520$ mm

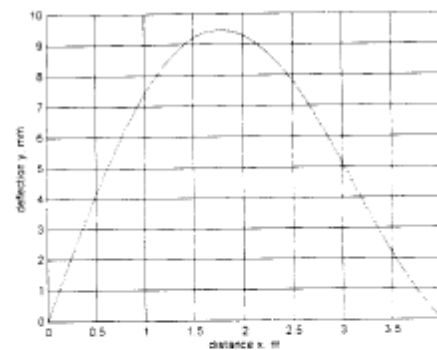


Fig.4: Deflection Curve: $y_{max}=9.47$ mm at $x=1779$ mm

The values of the functions at any point can now be read off from the above graphs. However if the exact values are required they can be obtained directly from the stored vectors representing the functions. If, for example, the exact deflection at a point E a distance $x=2440$ mm from the LHS is required then a statement $yE=y(2441)*1000$ would have to be included in the script before running it. For the data used above the result for $y(x)$ at E has been obtained as $yE=8.06$ mm.

4 CONCLUDING REMARKS

This paper has presented a software-based solution of to a simple undergraduate design and analysis problem, in the form of a MATLAB script that can be used to analyze and solve a beam-bending problem subjected to different loading conditions. The attached MATLAB script enhances the educational benefits to students by giving them a tool to play with as they visualize the effects of changing loading conditions on bending moments, reactions, shear force, bending stress and deflection, as well as their locations.

The solution of this typical mechanics problem showcases the general importance of the work reported in this paper: the use of interactive, editable software solutions to engineering problems. These should increasingly be explored to give students a more meaningful exploration-driven education.

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