CASCADE SYSTEM RELIABILITY WITH RAYLEIGH DISTRIBUTION

A. Rekha and V. C. Chenchu Raju
Faculty of Engineering and Technology
University of Botswana
Gaborone, Botswana

The stress-attenuation in cascade reliability has been studied when both stress and strength follow Rayleigh distribution. The expression for the reliability R(n), for a system to survive with the first (n – 1) components failed and the n^{th} component active, is obtained. R(n) is calculated numerically, for n = 1,2,3 and 4, for different values of the parameter ρ and the attenuation factor k. It has been found that with lower attenuation factors a high degree of reliability can be attained.

1 INTRODUCTION

The field of reliability has become an important factor in systems design and development as it analyses the out growth of problems with electronic systems. Thus, with the increasing sophistication and miniaturization of electronic and telemetric equipments needed for defence, space research problems, satellites, etc. demand a high degree of reliability. Hence, the evaluation of system reliability becomes an essential part to decide whether a system will accomplish its mission successfully.

The intuitive and redesign approach has made a way for an entirely new approach to reliability, one that is statistically defined, calculated and designed into a piece of equipment.

The probability of the failure of a system depends upon the stress and strength components of the system [1]. If the stress exceeds the strength, the system failure occurs. The stress applied will be of random magnitude with considerable variations due to imperfection in manufacturing process and non-uniformity in the materials of random character to the component's strength. Thus the strength of the component is also a random variable.

The concept of cascade reliability was considered by Srivastaw and Pandit [2]. Raghava Char et al [3] considered the reliability of an n-cascade system with stress attenuation for identical distributions. Uma Maheswari et al [4] considered the system reliability of an n-cascade system with stress and strength following normal and exponential distributions respectively. They have shown that a high degree of reliability is attained for lower values of stress and strength parameters. Similar stress attenuation studies with stress and strength following Gamma and Exponential distributions were studied by Rekha and Shyam Sunder [5]. They concluded that for higher parameter values and lower attenuation factors a high degree of reliability could be obtained. In the present paper the stress-attenuated cascade reliability is studied, for a system which works for a long time and starts wearing out, when both stress and strength are subjected to Rayleigh distribution as this distribution corresponds to the wearout failures [1].

2 MATHEMATICAL MODEL

Srivastaw and Pandit [2] defined the n-cascade system as a special type of standby system with n-components. Cascade redundancy is a hierarchical standby redundancy, where a standby component takes the place of the failed component with changed stress. This changed stress is k times the preceding stress, where k is the attenuation factor.

Let x_1 , x_2 , x_3 ,..., x_n be the strengths of the components c_1 , c_2 , c_3 ,..., c_n as arranged in order of activation. All the 'x_i's are independently distributed random variables with probability density function $f(x_i)$, i = 1,2,3,...,n. Also let y_1 be the stress on the first component which is also randomly distributed with the density function $g(y_1)$.

If $y_1 < x_1$, the first component c_1 works and hence the system survives. $y_1 > x_1$ leads to the failure of the first component c_1. Then the standby second component c_2 , takes its place which has a strength x_2. However, the stress y_2 on c_2 will normally be different from y_1 and $y_2 = k_2^* y_1$, where $k_2^* = k_1 k_2$, is the cumulative attenuation factor. Though the system has suffered a loss of one component, it survives if $y_2 < x_2$ and so on.

In general if the $(n-1)^{th}$ component c_{n-1} fails, the n^{th} component c_n with strength x_n gets activated and will be subjected to the stress $y_n = k_n^* y_1$. Thus the system could survive with a loss of first $(n-1)$ components if and only if $x_i < y_i$ (i = 1,2,3,...n – 1) and $x_n > y_n$. The reliability $R(n)$ of the system to survive with the first $(n-1)$ components failed and the n^{th} component active is given by:

$$R(n) = P\left[\left\{\bigcap_{i=1}^{n-1}(x_i < y_i)\right\} \cap (x_n > y_n)\right]$$

$$= P\left[\left\{\bigcap_{i=1}^{n-1}(x_i < k_i^* y_1)\right\} \cap (x_n > k_n^* y_1)\right]$$

$$= \int_{-\infty}^{\infty}\left[\left\{\int_{-\infty}^{k_1^* y_1} f(x_1)dx_1\right\}\left\{\int_{-\infty}^{k_2^* y_1} f(x_2)dx_2\right\}\ldots\right.$$

$$\left.\left\{\int_{-\infty}^{k_{n-1}^* y_1} f(x_{n-1})dx_{n-1}\right\}\left\{\int_{k_n^* y_1}^{\infty} f(x_n)dx_n\right\} g(y_1)\right] dy_1$$

$$= \int_{-\infty}^{\infty}\left[F_1(k_1^* y_1) F_2(k_2^* y_1) F_3(k_3^* y_1)\ldots\right.$$

$$\left.F_{n-1}(k_{n-1}^* y_1)\overline{F}_n(k_n^* y_1)\right] g(y_1)dy_1$$

where $F_i(k_i^* y_1) = \int_{-\infty}^{k_i^* y_1} f(x_i)dx_i$ is the cumulative distribution function and $\overline{F}_n$ is the complement of F_n.

The reliability of the system R_n is given by

$$R_n = \sum_{i=1}^{n} R(i)$$

Rayleigh Distributed Stress and Strength:

If the stress y_1 and strength x_i ($i = 1,2,3,\ldots,n$) follow Rayleigh distribution with the density functions

$$f(x_i) = \lambda_i x_i e^{-\lambda_i x_i^2/2}, \; 0 \le x_i < \infty \quad \lambda_i > 0$$

$$g(y_1) = \mu y_1 e^{-\mu y_1^2/2}, \; 0 \le y_1 < \infty \quad \mu > 0$$

$$R(n) = P\left[x_1 < k_1^* y_1, x_2 < k_2^* y_1, \ldots,\right.$$

$$\left.x_{n-1} < k_{n-1}^* y_1, x_n > k_n^* y_1\right]$$

$$= \int_0^{\infty}\left[F_1(k_1^* y_1) F_2(k_2^* y_1) F_3(k_3^* y_1)\ldots\right.$$

$$\left.F_{n-1}(k_{n-1}^* y_1)\overline{F}_n(k_n^* y_1)\right] g(y_1)dy_1$$

where $F_i(k_i^* y_1) = \int_0^{k_i^* y_1} f(x_i)dx_i$

$$= \int_0^{k_i^* y_1} \lambda_i x_i e^{-\lambda_i x_i^2/2} dx_i$$

$$= 1 - e^{-\lambda_i (k_i^* y_1)^2/2}$$

Then

$$R(n) = \int_0^{\infty}\left[\left\{1 - e^{-\lambda_1 (k_1^* y_1)^2/2}\right\}\right.$$

$$\times\left\{1 - e^{-\lambda_2 (k_2^* y_1)^2/2}\right\}\ldots\left\{1 - e^{-\lambda_{n-1} (k_{n-1}^* y_1)^2/2}\right\}$$

$$\left.\times e^{-\lambda_n (k_n^* y_1)^2/2}\right] \mu y_1 e^{-\mu y_1^2/2} dy_1$$

$$= \frac{\mu}{\mu + \lambda_n k_n^{*2}} - \sum_{i=1}^{n-1} \frac{\mu}{\lambda_i k_i^{*2} + \lambda_n k_n^{*2} + \mu}$$

$$+ \sum_{i<j=1}^{n-1} \frac{\mu}{\lambda_i k_i^{*2} + \lambda_j k_j^{*2} + \lambda_n k_n^{*2} + \mu} -$$

$$\ldots(-1)^{n-1} \frac{\mu}{\lambda_1 k_1^{*2} + \lambda_2 k_2^{*2} + \ldots + \lambda_n k_n^{*2} + \mu}$$

Let $\rho_i = \dfrac{\lambda_i}{\mu}$. Then

$$R(n) = \frac{1}{1 + \rho_n k_n^{*2}} - \sum_{i=1}^{n-1} \frac{1}{\rho_i k_i^{*2} + \rho_n k_n^{*2} + 1}$$

$$+ \sum_{i<j=1}^{n-1} \frac{1}{\rho_i k_i^{*2} + \rho_j k_j^{*2} + \rho_n k_n^{*2} + 1} -$$

$$\ldots(-1)^{n-1} \frac{1}{\rho_1 k_1^{*2} + \rho_2 k_2^{*2} + \ldots + \rho_n k_n^{*2} + 1}$$

from which

$$R(1) = \frac{1}{1 + \rho_1 k_1^{*2}}$$

$$R(2) = \frac{1}{1+\rho_2 k_2^{*2}}$$

$$- \frac{1}{\rho_1 k_1^{*2} + \rho_2 k_2^{*2} + 1}$$

$$R(3) = \frac{1}{1+\rho_3 k_3^{*2}}$$

$$- \frac{1}{\rho_1 k_1^{*2} + \rho_3 k_3^{*2} + 1}$$

$$- \frac{1}{\rho_2 k_2^{*2} + \rho_3 k_3^{*2} + 1}$$

$$+ \frac{1}{\rho_1 k_1^{*2} + \rho_2 k_2^{*2} + \rho_3 k_3^{*2} + 1}$$

$$R(4) = \frac{1}{1+\rho_4 k_4^{*2}}$$

$$- \frac{1}{\rho_3 k_3^{*2} + \rho_4 k_4^{*2} + 1}$$

$$- \frac{1}{\rho_2 k_2^{*2} + \rho_4 k_4^{*2} + 1}$$

$$- \frac{1}{\rho_1 k_1^{*2} + \rho_4 k_4^{*2} + 1}$$

$$+ \frac{1}{\rho_1 k_1^{*2} + \rho_2 k_2^{*2} + \rho_4 k_4^{*2} + 1}$$

$$+ \frac{1}{\rho_1 k_1^{*2} + \rho_3 k_3^{*2} + \rho_4 k_4^{*2} + 1}$$

$$+ \frac{1}{\rho_2 k_2^{*2} + \rho_3 k_3^{*2} + \rho_4 k_4^{*2} + 1}$$

$$- \frac{1}{\rho_1 k_1^{*2} + \rho_2 k_2^{*2} + \rho_3 k_3^{*2} + \rho_4 k_4^{*2} + 1}$$

3 CONCLUSION

The values of $R(n)$ for $n = 1,2,3$ and 4 are evaluated numerically, for $\rho_i = \rho$ taking values from 0.5 to 10 while the cumulative attenuation factor k_i^* takes four different values such as $i!$, $(i+1)!$, $1/i!$ and $1/(1+i)!$, and tabulated in the tables 1 to 4. Graphs of R_n were drawn for the cases where $k_i^* = 1/i!$ and $1/(1+i)!$ only as the values in the other two cases are close to each other. It has been found that for lower attenuation factors a high degree of reliability can be attained even though Rayleigh distribution finds application in reliability only when the components are characterized with linearly increasing failure rates.

4 ACKNOWLEDGMENTS

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5 REFERENCES

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Table 1 (for $k_i^* = i!$)

ρ	R(1)	R(2)	R(3)	R(4)	R_2	R_3	R_4
0.5	0.6667	0.0476	0.0002	0.0000	0.7143	0.7145	0.7145
1.0	0.5000	0.0333	0.0001	0.0000	0.5333	0.5335	0.5335
1.5	0.4000	0.0252	0.0001	0.0000	0.4252	0.4253	0.4253
2.0	0.3333	0.0202	0.0001	0.0000	0.3535	0.3536	0.3536
2.5	0.2857	0.0168	0.0001	0.0000	0.3025	0.3026	0.3026
3.0	0.2500	0.0144	0.0000	0.0000	0.2644	0.2645	0.2645
3.5	0.2222	0.0126	0.0000	0.0000	0.2348	0.2349	0.2349
4.0	0.2000	0.0112	0.0000	0.0000	0.2112	0.2112	0.2112
4.5	0.1818	0.0101	0.0000	0.0000	0.1919	0.1919	0.1919
5.0	0.1667	0.0092	0.0000	0.0000	0.1758	0.1759	0.1759
5.5	0.1538	0.0084	0.0000	0.0000	0.1622	0.1623	0.1623
6.0	0.1429	0.0077	0.0000	0.0000	0.1506	0.1506	0.1506
6.5	0.1333	0.0072	0.0000	0.0000	0.1405	0.1405	0.1405
7.0	0.1250	0.0067	0.0000	0.0000	0.1317	0.1317	0.1317
7.5	0.1176	0.0063	0.0000	0.0000	0.1239	0.1239	0.1239
8.0	0.1111	0.0059	0.0000	0.0000	0.1170	0.1170	0.1170
8.5	0.1053	0.0056	0.0000	0.0000	0.1108	0.1109	0.1109
9.0	0.1000	0.0053	0.0000	0.0000	0.1053	0.1053	0.1053
9.5	0.0952	0.0050	0.0000	0.0000	0.1003	0.1003	0.1003
10.0	0.0909	0.0048	0.0000	0.0000	0.0957	0.0957	0.0957

Table 2 (for $k_i^* = (1+i)!$)

ρ	R(1)	R(2)	R(3)	R(4)	R_2	R_3	R_4
0.5	0.3333	0.0050	0.0000	0.0000	0.3383	0.3383	0.3383
1.0	0.2000	0.0026	0.0000	0.0000	0.2026	0.2026	0.2026
1.5	0.1429	0.0018	0.0000	0.0000	0.1446	0.1446	0.1446
2.0	0.1111	0.0014	0.0000	0.0000	0.1125	0.1125	0.1125
2.5	0.0909	0.0011	0.0000	0.0000	0.0920	0.0920	0.0920
3.0	0.0769	0.0009	0.0000	0.0000	0.0778	0.0778	0.0778
3.5	0.0667	0.0008	0.0000	0.0000	0.0674	0.0674	0.0674
4.0	0.0588	0.0007	0.0000	0.0000	0.0595	0.0595	0.0595
4.5	0.0526	0.0006	0.0000	0.0000	0.0532	0.0532	0.0532
5.0	0.0476	0.0005	0.0000	0.0000	0.0482	0.0482	0.0482
5.5	0.0435	0.0005	0.0000	0.0000	0.0440	0.0440	0.0440
6.0	0.0400	0.0005	0.0000	0.0000	0.0405	0.0405	0.0405
6.5	0.0370	0.0004	0.0000	0.0000	0.0375	0.0375	0.0375
7.0	0.0345	0.0004	0.0000	0.0000	0.0349	0.0349	0.0349
7.5	0.0323	0.0004	0.0000	0.0000	0.0326	0.0326	0.0326
8.0	0.0303	0.0003	0.0000	0.0000	0.0306	0.0306	0.0306
8.5	0.0286	0.0003	0.0000	0.0000	0.0289	0.0289	0.0289
9.0	0.0270	0.0003	0.0000	0.0000	0.0273	0.0273	0.0273
9.5	0.0256	0.0003	0.0000	0.0000	0.0259	0.0259	0.0259
10.0	0.0244	0.0003	0.0000	0.0000	0.0247	0.0247	0.0247

Table 3 (for $k_i^* = 1/i!$)

ρ	R(1)	R(2)	R(3)	R(4)	R_2	R_3	R_4
0.5	0.6667	0.2735	0.0579	0.0020	0.9402	0.9980	1.0000
1.0	0.5000	0.3556	0.1362	0.0082	0.8556	0.9918	1.0000
1.5	0.4000	0.3794	0.2035	0.0169	0.7794	0.9830	0.9999
2.0	0.3333	0.3810	0.2585	0.0270	0.7143	0.9728	0.9998
2.5	0.2857	0.3730	0.3032	0.0378	0.6587	0.9618	0.9996
3.0	0.2500	0.3609	0.3396	0.0489	0.6109	0.9505	0.9994
3.5	0.2222	0.3473	0.3696	0.0601	0.5695	0.9391	0.9992
4.0	0.2000	0.3333	0.3943	0.0712	0.5333	0.9276	0.9989
4.5	0.1818	0.3196	0.4148	0.0823	0.5015	0.9163	0.9986
5.0	0.1667	0.3065	0.4319	0.0932	0.4732	0.9051	0.9982
5.5	0.1538	0.2941	0.4461	0.1038	0.4479	0.8940	0.9979
6.0	0.1429	0.2824	0.4580	0.1143	0.4252	0.8832	0.9975
6.5	0.1333	0.2714	0.4679	0.1245	0.4047	0.8726	0.9971
7.0	0.1250	0.2611	0.4761	0.1344	0.3861	0.8622	0.9966
7.5	0.1176	0.2514	0.4829	0.1442	0.3691	0.8520	0.9962
8.0	0.1111	0.2424	0.4885	0.1537	0.3535	0.8420	0.9957
8.5	0.1053	0.2340	0.4931	0.1629	0.3392	0.8323	0.9952
9.0	0.1000	0.2261	0.4967	0.1719	0.3261	0.8228	0.9947
9.5	0.0952	0.2186	0.4996	0.1807	0.3139	0.8135	0.9942
10.0	0.0909	0.2116	0.5018	0.1893	0.3025	0.8044	0.9936

Table 4 (for $k_i^* = 1/(1+i)!$)

ρ	R(1)	R(2)	R(3)	R(4)	R_2	R_3	R_4
0.5	0.8889	0.1083	0.0029	0.0000	0.9971	1.0000	1.0000
1.0	0.8000	0.1904	0.0096	0.0000	0.9904	1.0000	1.0000
1.5	0.7273	0.2541	0.0185	0.0001	0.9814	0.9999	1.0000
2.0	0.6667	0.3045	0.0286	0.0002	0.9712	0.9998	1.0000
2.5	0.6154	0.3449	0.0393	0.0004	0.9603	0.9996	1.0000
3.0	0.5714	0.3776	0.0503	0.0006	0.9491	0.9994	1.0000
3.5	0.5333	0.4044	0.0615	0.0009	0.9377	0.9991	1.0000
4.0	0.5000	0.4263	0.0726	0.0011	0.9263	0.9989	1.0000
4.5	0.4706	0.4444	0.0835	0.0014	0.9150	0.9986	1.0000
5.0	0.4444	0.4594	0.0943	0.0018	0.9039	0.9982	1.0000
5.5	0.4211	0.4719	0.1049	0.0021	0.8929	0.9979	1.0000
6.0	0.4000	0.4821	0.1153	0.0025	0.8821	0.9975	1.0000
6.5	0.3810	0.4906	0.1255	0.0029	0.8716	0.9971	1.0000
7.0	0.3636	0.4976	0.1354	0.0034	0.8612	0.9966	1.0000
7.5	0.3478	0.5033	0.1451	0.0038	0.8511	0.9962	1.0000
8.0	0.3333	0.5078	0.1545	0.0043	0.8412	0.9957	1.0000
8.5	0.3200	0.5115	0.1637	0.0048	0.8315	0.9952	1.0000
9.0	0.3077	0.5143	0.1727	0.0053	0.8220	0.9947	1.0000
9.5	0.2963	0.5164	0.1815	0.0058	0.8127	0.9942	1.0000
10.0	0.2857	0.5179	0.1900	0.0064	0.8036	0.9936	1.0000

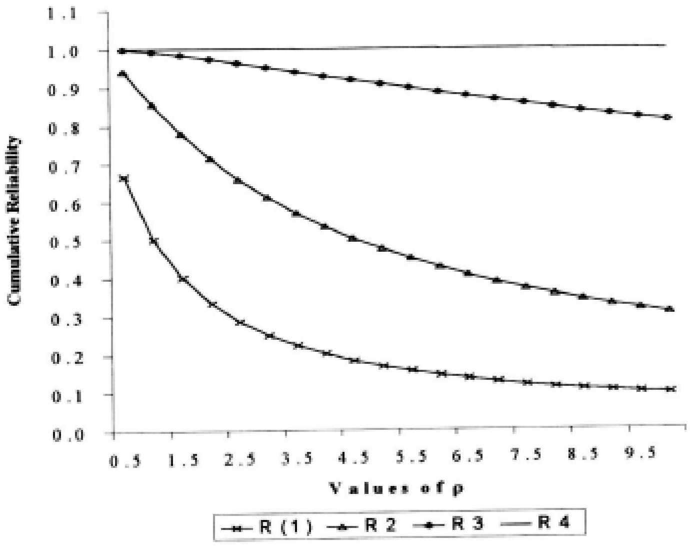

Fig.1 Cumulative Reliability versus ρ for $k_i^* = 1/i!$

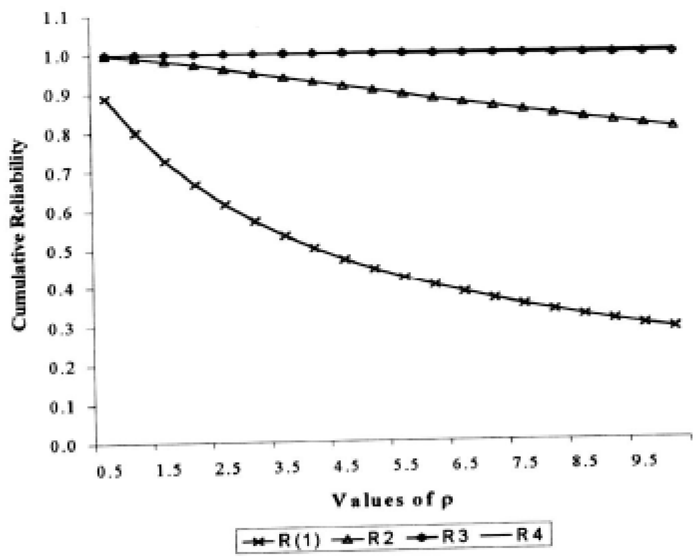

Fig.2 Cumulative Reliability versus ρ for $k_i^* = 1/(i+1)!$