

ON THE RIESZ REPRESENTATION THEOREM AND INTEGRAL OPERATORS

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ABSTRACT. We present a Riesz representation theorem in the setting of extended integration theory as introduced in [6]. The result is used to obtain boundedness theorems for integral operators in the more general setting of spaces of vector valued extended integrable functions.

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1. Introduction. An extended notion of integrability was recently introduced in [6]. Many fundamental properties of the Lebesgue-Bochner integration theory such as Fatou's Lemma, Lebesgue monotone convergence and dominated convergence theorems, naturally generalize to such a new setting of the integral. The extension of scalar version of the Lebesgue-Nikodým theorem to the vector valued case was also established in [8]. Another important property of the Lebesgue-Bochner integration theory, known as the Riesz representation theorem, gives characterization of the dual of the spaces of p -Bochner-integrable functions. Such theorem plays crucial role in the study of boundedness of some classes of operators between function spaces. For instance, many authors (see e.g. [3, 9]) have proved some boundedness theorems for the so-called integral operators $T : L^p(\Omega, V, \mu) \rightarrow L^q(\Omega', W, \mu')$ of the form

$$(1.1) \quad Tf(\cdot) = \int_{\Omega} \kappa(\cdot, \omega)[f(\omega)]d\mu(\omega)$$

where the kernel κ is a function taking value in the space $\mathcal{L}(V, W)$ of bounded linear operators from a Banach space V to a Banach space W . The study of such kind of operators is at the heart of many practical applications: for instance in solving inhomogeneous differential and integral equations, in the theory of evolutionary equations. For details on integral operators with scalar valued kernels and its applications, the reader is referred to [2], and [1].

As in many areas of mathematics, it is always desirable and useful to have at our disposal a theory at a level of generality that will allow as wide of a spectrum of applications as possible. This note presents a new extended version of the Riesz representation theorem that will allow some extension of boundedness theorems for integral operators.