

Bivariate models: relationships between solar irradiation and either sunshine duration or extremum temperatures

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Abstract

The Republic of Botswana is one of the sunniest countries in Southern Africa. It has very little cloud cover, insufficient rainfall, very low humidity, and very low wind speed throughout the year for most parts of the country. The daily extremum temperatures appear to be very much related to solar irradiation which in turn depends on sunshine duration. In Botswana, solar irradiation on a horizontal surface is measured only at Sebele, but sunshine duration and extremum temperatures are measured at several locations throughout the country. This paper presents bivariate models that relate solar irradiation to sunshine duration, and solar irradiation to extremum temperatures for Sebele, Botswana. Autocorrelation analysis revealed that the solar irradiation series is stationary for $d=2$ and $D=0$, sunshine duration series is stationary for $d=0$ and $D=0$, while the extremum temperatures series are stationary for either $d=0$ and $D=N$ where $N=1, 2, \dots$ or $d=1$ and $D=1$. It is found that there is a lag of three months between the peaks of the differenced series of fractional sunshine duration and fractional solar irradiation. On the other hand it is found that there is at most a lag of one month between the peaks of the differenced series of maximum temperature and solar irradiation, and that there is no lag between the peaks of the differenced series of minimum temperature and solar irradiation. Analysis of the noise component revealed that the bivariate processes under consideration behaved either as a purely seasonal MA processes of order (0,1,1) or as ARIMA processes of order (0, 1, 1)×(0, 1, 1)₁₂ or as a purely nonseasonal, autoregressive process of

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order 2. We claim that the relationships found for Sebele can be applied to estimate solar irradiation at other locations with climatic conditions similar to Botswana.

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1. Introduction

Solar radiation measurement and modeling are amongst the important aspects of solar energy applications as far as the sizing of solar devices is concerned. In developing countries it is usually the extremum temperatures that are measured for most locations, albeit sunshine duration is measured at few locations and solar irradiation is measured at even fewer locations. In that case, one resorts to modeling to estimate solar irradiation at locations for which measured solar irradiation data is not available. Ångström type relationships are the most extensively used to estimate solar irradiation from sunshine duration [1]. Whereas solar irradiation does depend directly on sunshine duration, it is also known to be related to other meteorological parameters such as the extremum temperatures, cloud cover, rainfall, etc. In Botswana, where the climate is mostly clear, with very little cloud cover, very low humidity and insufficient rainfall, the extremum temperatures appear to be very much determined from solar irradiation. Consequently, understanding the relationship between extremum temperatures and solar irradiation may provide an alternative approach to estimating solar irradiation for locations with climatic conditions similar to Botswana, where the measured temperatures data are available for a large number of locations and for longer durations than the measured data for sunshine duration. In this paper, relationships between fractional sunshine duration (sunshine duration as a fraction of the day length) and fractional solar irradiation (terrestrial solar irradiation as a fraction of the extraterrestrial solar irradiation) on a horizontal surface, and that between mean monthly extremum temperatures and mean monthly solar irradiation on a horizontal surface for Sebele, Botswana, are studied using bivariate autoregressive integrated moving average (ARIMA) modeling techniques.

2. Sunshine and temperatures data for Sebele, Botswana

Sebele (latitude: 24° 34' S; longitude: 25° 57' E; altitude 994 m), 10 km north-east of the capital city Gaborone is the earliest site in Botswana where measurements of solar radiation data was started in September/October 1975 by the Department of Agricultural Research, Ministry of Agriculture, Botswana, and to date remains the only site in Botswana for which both daily sunshine duration and solar irradiation on a horizontal surface are available. Sunshine duration, on the other hand, is measured at geographically well-distributed 14 synoptic stations throughout the country by the Department of Meteorological Services. Sunshine duration is measured using the Campbell–Stokes recorder, and the daily global irradiation is measured using the Kipp and Zonnen pyranometer. The pyranometer was calibrated annually against a standard

precision pyranometer following the IGY procedure [2] under clear skies conditions. The daily sunshine duration data available is up to date, with only about 2% of data points missing. Solar irradiation data on the other hand, due to failure of the measuring instruments, has about 5% data points missing for the period 1975 to 1992, has been measured for just a few months during 1993 to 1994, and has not been measured beyond 1994. With an average of 314 sunny days, and 3320 hours of sunshine per year [3], Sebele is a good representative of the solar conditions in parts of Southern Africa such as Botswana, Namibia and the north-west portion of South Africa.

Climatically, Botswana is arid to semi-arid, and experiences extremes of temperatures. During the peak summer months (December/January), the day-temperatures in some locations may occasionally reach as high as 45°C, whereas the peak winter-nights (June/July) experience occasional below-freezing temperatures resulting in night frost. The extremum temperatures data for most locations of interest in Botswana are available for longer periods than the solar radiation data, some records including Sebele extend as far back as the mid 60s, and have less than 1% missing data points. For this study we have used data for a period of seventeen years from 1976 to 1992 for which the mean monthly values for all four sets of data, namely sunshine duration, solar irradiation, maximum temperatures and minimum temperatures are available with less than 2% data missing. Figure 1 shows the mean monthly extremum temperatures for Sebele. The shape of these graphs are very similar to the shape of the monthly average solar irradiation graphs in Jain and Lungu [4]. This suggests a strong relationship between solar irradiation and extremum temperatures.

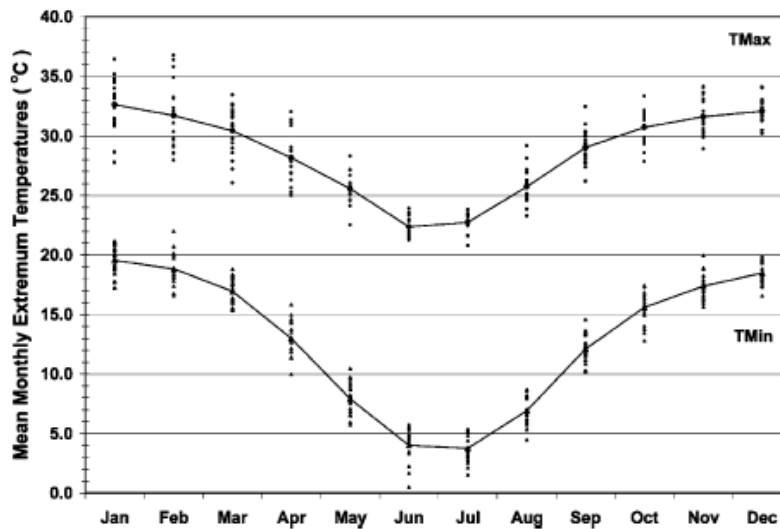


Fig. 1. Mean monthly extremum temperatures (1976–1992) for Sebele, Botswana.

3. Characteristics of extremum temperatures for Sebele

The characteristics of the mean monthly solar irradiation and sunshine duration for Sebele are discussed in Jain and Lungu [4]. Subjecting the monthly averages extremum temperatures series M_t , and the total extremum temperatures series $Y_{\alpha,t}$ to harmonic analysis as in Jain and Lungu [4] reveals that: (i) for the monthly averages series the first two harmonics are the most significant as they explain about 91% of the variance of the M_t series of extremum temperatures. (ii) For the total series of maximum temperatures the 25th and the 50th harmonics explain about 80% of the variance. On the other hand, the minimum temperatures are more deterministic as two harmonics (25th and the 50th) explain 95% of the variance of the total series. (iii) It is evident from these results that the maximum temperatures are in general more stochastic in nature as 20% of the variance is explained by this component.

3.1. Stochastic components of extremum temperatures

Following Jain and Lungu [4], several autocorrelations for extremum temperatures were calculated for a number of possible differencing schemes $d=0, 1, 2$, and $D=0, 1, 2$. Two possible schemes for further investigation suggested by these autocorrelations are $d=0$ and $D=N$ where $N=1, 2, \dots$, and $d=1$ and $D=1$.

The model $d=0$ and $D=N$ is purely seasonal. A detailed discussion for a specific example for $d=0, D=1$ follows: The autocorrelation functions and corresponding partial autocorrelation functions for this case are given in Fig. 2. The autocorrelation functions display seasonality as evidenced by the large $|r_i|$ values at $r_1, r_{11}, r_{12}, r_{13}$. Furthermore, compared to the 95% confidence limits, most of the autocorrelations at other lags are insignificant. These autocorrelation functions mimic the behaviour of a moving average process. This is confirmed by the partial autocorrelations which

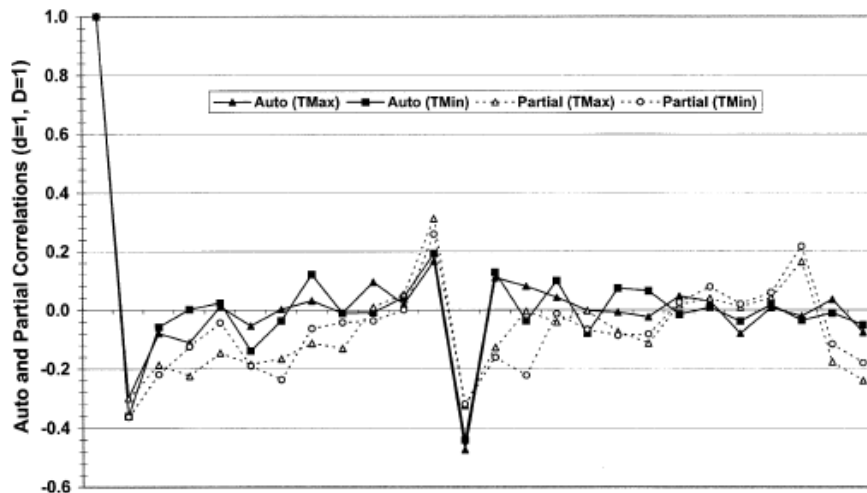


Fig. 2. Auto and partial correlations for the (0, 1) model of the mean monthly extremum temperatures.

Table 1
Model parameters and statistics for mean monthly fractional sunshine duration, and extremum temperatures input series for the Bivariate model

Input X_t	Autoregressive parameters				Model	Portmanteau statistics	Critical value of portmanteau statistics
	θ_1	θ_2	θ_3	Σa_i^2			
FSD (2, 0)	0.35	0.20	–	–	AR(2)	21.6	35
T_{Max} (0, 1)	–	–	0.75	1012	$(0,1,1)_{12}$	20.9	35
T_{Max} (1, 1)	0.65	–	0.85	834	$(0,1,1) \times (0,1,1)_{12}$	14.1	35
T_{Min} (0, 1)	–	–	0.70	510	$(0,1,1)_{12}$	24.3	35
T_{Min} (1, 1)	0.75	–	0.80	409	$(0,1,1) \times (0,1,1)_{12}$	14.0	35

mimic autoregressive behaviour. The seasonality of the model is supported by the fact that the significance of autocorrelations $r_1, r_{11}, r_{12}, r_{13}$ is removed by determining the parameter Θ_1 only, and the behaviour of the time series is represented by a model of the form:

$$\nabla_{12}Z_t = (1 - \Theta_1 B^{12})a_t \tag{1}$$

The values of the parameter Θ_1 that minimize Σa_i^2 for the extremum temperature series are given in Table 1, and the residual autocorrelations are shown in Fig. 3. It is evident that compared to 95% confidence limits the residuals are representative of a random series.

For $d=1, D=1$ the autocorrelations together with corresponding partial autocorre-

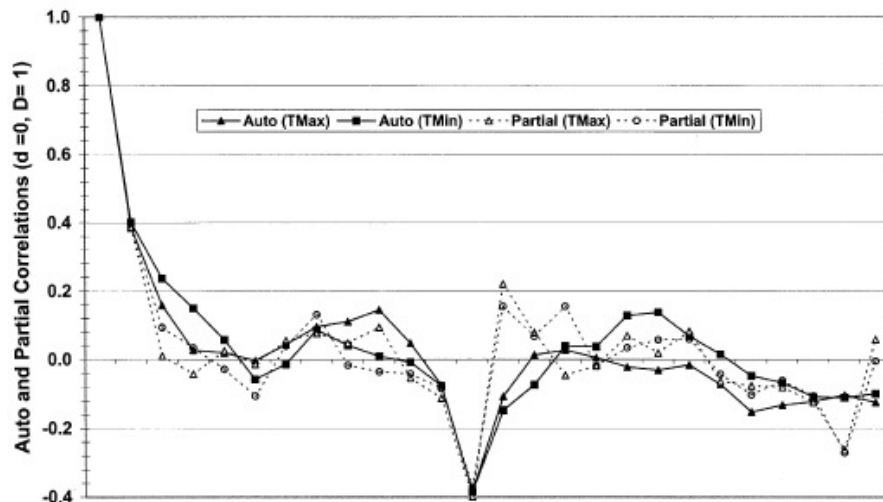


Fig. 3. Autocorrelations of the white noise (residues) for the extremum temperatures models.

lations are shown in Fig. 4. The autocorrelations are all insignificant except r_1 , r_{11} , r_{12} , r_{13} , and in this case two parameters θ_1 and Θ_1 are required to remove their significance. This is an indication of the presence of seasonal as well as non-seasonal effects. The autocorrelations together with the partial autocorrelations indicate that a model of the form $(0, 1, 1) \times (0, 1, 1)_{12}$ could be fitted to the extremum temperatures data. The proposed model is:

$$\nabla \nabla_{12} Z_t = (1 - \theta_1 B)(1 - \Theta_1 B^{12}) a_t \quad (2)$$

The admissible values of the parameters θ_1 and Θ_1 must satisfy $-1 < \theta_1 < 1$, and $-1 < \Theta_1 < 1$. The initial values of θ_1 and Θ_1 were estimated using:

$$r_1 = -\left(\frac{\theta_1}{1 + \theta_1^2}\right), \text{ and } r_{12} = -\left(\frac{\Theta_1}{1 + \Theta_1^2}\right), \quad (3)$$

where r_1 and r_{12} are the autocorrelations at lags 1 and 12, respectively. The appropriate values of θ_1 and Θ_1 are those that minimize $\sum a_t^2$. The minimization procedure is carried out by plotting the sum of squares-surface for a range of values of θ_1 and Θ_1 . The surface is well behaved, having just one minimum for each case. Table 1 gives the values of the optimum parameters.

The residual autocorrelations for the seasonal, non-seasonal multiplicative Integrated Moving Average (IMA) model $(0, 1, 1) \times (0, 1, 1)_{12}$ are also shown in Fig. 3. These autocorrelations are typical of a random series. Note also that the seasonal effects in the residual autocorrelations have been nearly eliminated as the autocorrelations r_{11} , r_{12} , r_{13} , r_{23} , r_{24} , r_{25} are all close to zero. The a_t sequences pass the portmanteau statistics test with the portmanteau statistics computed using the first 25 values of the residual autocorrelations. Compared with the value of χ^2 at the 5% level with 23 degrees of freedom all the autocorrelations are insignificant, suggesting

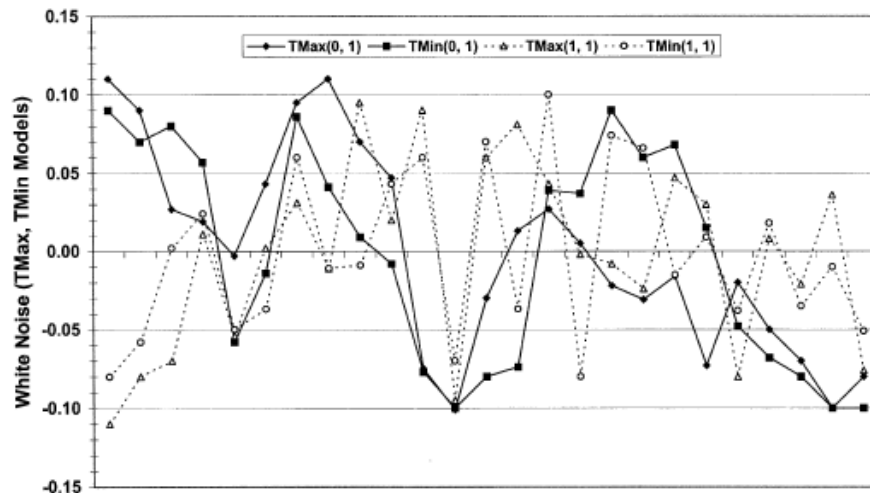


Fig. 4. Auto and partial correlations for the (1, 1) model of the mean monthly extremum temperatures.

further that the residual series may be considered as random (Table 1). The two models, namely the purely seasonal model $(0, 1, 1)_{12}$, and the IMA model $(0, 1, 1) \times (0, 1, 1)_{12}$, can be considered as suitable representatives of the given extremum temperatures series.

4. Bivariate modeling

4.1. Correlation analysis and objectives of the study

To establish the objectives of the study, we begin by investigating the relationships between mean monthly solar irradiation, sunshine duration, maximum temperatures (T_{Max}) and minimum temperatures (T_{Min}), and between fractional solar irradiation (\bar{H}/\bar{H}_0) and fractional sunshine duration (\bar{n}/\bar{N}) for Sebele where \bar{H} is the monthly average solar irradiation measured on a horizontal surface, \bar{H}_0 is the monthly average extraterrestrial solar irradiation [5] represented by the Julian day value for the month, \bar{n} is the monthly average sunshine duration, \bar{N} and is day-length [5] for the Julian-day of the month. The correlation matrix for these variables is given in Table 2.

From Table 2, it is evident that solar irradiation is highly correlated to both the maximum and the minimum temperatures. Furthermore, fractional solar irradiation is found to be strongly correlated to fractional sunshine duration. Therefore, in this study our objective shall be to determine the bivariate models that represent relationships between solar irradiation and maximum temperatures, solar irradiation and minimum temperatures, and fractional solar irradiation and fractional sunshine duration. Extremum temperatures (T_{Max} and T_{Min}) are also found to be highly correlated to each other, but their relationship is not investigated further.

4.2. Theory of linear transfer functions

Suppose there are N meteorological observations on two variables, X_t (independent variable) and Y_t (dependent variable) at equispaced intervals of time t (monthly averages in this case). These observations may be denoted by $(X_1, Y_1), (X_2, Y_2), \dots, (X_N, Y_N)$, and may also be regarded as a finite realization of a discrete bivariate process with the X_t series regarded as the input, and the Y_t series as the output. One needs to find the impulse response function $\{v_k\}$, where $k=0, 1, \dots$, of the system:

Table 2
Correlation matrix for the mean monthly solar irradiation, sunshine duration, maximum temperatures (T_{Max}) and minimum temperatures (T_{Min}) for Sebele

	Irradiation	Duration	T_{Max}	T_{Min}
Irradiation	1	0.8079 ^a	0.8413	0.7826
Duration		1	0.1748	0.1112
T_{Max}			1	0.9175
T_{Min}				1

^a This correlation is between the fractional solar irradiation, and fractional sunshine duration.

$$Y_t = v(B)X_{t-b} \quad (4)$$

where $v(B) = v_0 - v_1B - v_2B^2 - \dots$ is called the transfer function, B is the backward shift operator and b is the delay parameter. Denoting the incremental changes in X and Y by:

$$x_t = \nabla^d X_t, \quad y_t = \nabla^d Y_t, \quad (5)$$

where d is the degree of differencing, $\nabla = (1-B)$, and for any series $\{Z_t\}$, $Z_{t-b} = B^b Z_t$, then it can be shown, on differencing Eq. (4), that x_t and y_t satisfy the same transfer function model as do X_t and Y_t , i.e.,

$$y_t = v(B)x_{t-b}. \quad (6)$$

The linear filter Eq. (6) may also be written in an alternative form which in general requires fewer parameters [6], i.e.,

$$\delta(B)y_t = \omega(B)x_{t-b} \quad (7)$$

where

$$\begin{aligned} \delta(B) &= 1 - \delta_1 B - \delta_2 B^2 - \dots - \delta_r B^r \\ \omega(B) &= \omega_0 - \omega_1 B - \omega_2 B^2 - \dots - \omega_s B^s \end{aligned} \quad (8)$$

On comparing Eqs (6) and (7), one obtains the identity:

$$v(B) = \delta^{-1}(B)\omega(B) \quad (9)$$

$$\begin{aligned} v_j &= 0, & j < b \\ v_j &= \delta_1 v_{j-1} + \delta_2 v_{j-2} + \dots + \delta_r v_{j-r} - \omega_0, & j = 0, 1, \dots, b \\ v_j &= \delta_1 v_{j-1} + \delta_2 v_{j-2} + \dots + \delta_r v_{j-r} - \omega_{j-b}, & j = (b+1), \dots, (b+s) \\ v_j &= \delta_1 v_{j-1} + \delta_2 v_{j-2} + \dots + \delta_r v_{j-r}, & j > (b+s). \end{aligned} \quad (10)$$

It may be noted that a plot of the weights v_k , $k=0, 1, \dots$ against lag k provides a pictorial representation of the impulse response function. In practice, however, the system is infected by disturbances or noise, whose net effect is to corrupt the output predicted by the transfer function model by an amount n_t so that the combined transfer function–noise model may now be written as [6]:

$$y_t = \delta^{-1}(B)\omega(B)x_{t-b} + n_t \quad (11)$$

where x_t and y_t are stationary series for some value of differencing d . The Box and Jenkins [6] pre-whitening procedure involves fitting an ARIMA model to the (differenced) input series as a first step. Suppose for a while that this is known as:

$$\phi(B)x_t = \theta(B)\alpha_t \quad (12)$$

where α_t denotes a pure random process. It is evident then, from Eq. (11) that the transformation $\phi(B)\theta^{-1}(B)$ transforms the correlated input series x_t to the uncorrelated white noise series α_t , i.e.,

$$\alpha_t = \phi(B)\theta^{-1}(B)x_t. \quad (13)$$

Suppose, further, that one can apply the same transformation to the output series, to give:

$$\beta_t = \phi(B)\theta^{-1}(B)y_t \tag{14}$$

and then calculate the cross-covariance function of the filtered input and output, namely α_t and β_t . It turns out that operating on both sides of Eq. (12) with $\phi(B)\theta^{-1}(B)$ yields:

$$\beta_t = v(B)\alpha_t + \varepsilon_t \tag{15}$$

where $\varepsilon_t = \phi(B)\theta^{-1}(B)n_t$ is the transformed noise series. Multiplying both sides of Eq. (15) by α_{t-k} and taking expectations, noting that α_t and n_t are uncorrelated gives:

$$v_k = \frac{\gamma_{\alpha\beta}(k)S_\beta}{S_\alpha} \tag{16}$$

where S_α^2 and S_β^2 are the variances of α_t and β_t respectively, and $\gamma_{\alpha\beta}(k) = E[\alpha_{1-b}\beta_t]$ is the cross-covariance function at lag k . The estimates v_k determined as outlined above are found to be reliable [6] and are used in this paper as a basis for estimating the parameters b , r , and s . In addition to identifying the orders r and s of operators $\delta(B)$ and $\omega(B)$ of Eq. (11), one seeks to identify appropriate ARIMA models that describe the noise at the output.

5. Parameter estimation, model identification and fitting

We seek to determine bivariate models with (i) monthly average fractional sunshine duration as input and fractional solar irradiation as output, and (ii) monthly average extremum temperatures as input and solar irradiation as output.

The order of monthly differencing d , the order of seasonal differencing D , the order of autoregressive operator $\theta(B)$ and the order of the moving average operator $\Theta(B)$ are determined by autocorrelation and partial autocorrelation analysis [4,6]. The values of d and D for which the input and output series are stationary are given in Table 3. It can be shown that the (differenced) input series x_t can be transformed

Table 3
Degrees of differencing d and D for which the input and output series are stationary

Input X_t	(d, D)	Output Y_t	(d, D)
Fractional sunshine duration	(0, 0)	Fractional solar irradiation	(2, 0)
Maximum temperature	(1, 1)	Solar irradiation	(2, 0)
	(0, 1)		(2, 0)
	(0, 2)		(2, 0)
Minimum temperature	(1, 1)	Solar irradiation	(2, 0)
	(0, 1)		(2, 0)
	(0, 2)		(2, 0)

into a purely random process α_t for values of the autoregressive parameters θ_1 , θ_2 and Θ_1 given in Table 1.

The autoregressive parameters obtained in Table 1 were applied to the corresponding output series y_t as shown in Table 3 to obtain the β_t series. Then the cross-correlation function $y_{\alpha\beta}(b)$ between the α_t and β_t series and the impulse response weights v_k were calculated and compared with their approximate standard errors. Cross-correlation functions for some α_t and β_t series, which are typical of those obtained for other values of d and D for the input/output series, are shown in Table 4.

It is evident from Table 4 that there is a delay of three months between the peak of differenced FSD and the peak of differenced FSI. On the other hand there is either no lag or a lag of one month between the peaks of the differenced maximum temperature and solar irradiation, and there is no lag between the peaks of the differenced minimum temperature and solar irradiation.

Next attention is given to identifying, estimating and fitting a model to the n_t series as in Box and Jenkins [6]. According to this procedure, it is assumed that the n_t series can be represented by a seasonal autoregressive integrated moving average process (ARIMA) of the type:

$$\Phi_P(B^s)\phi_p(B)\nabla_S^D\nabla^d n_t = \Theta_Q(B^S)\theta_q(B)a_t \quad (17)$$

where (p, d, q) are orders of a purely nonseasonal model, $(P, Q, D)_{12}$ are the orders of a purely seasonal model, $(p, d, q) \times (P, D, Q)_{12}$ are orders of the nonseasonal–seasonal model, and a_t is a white noise series. Values for the parameters in Eq. (17) were determined by autocorrelation and partial-autocorrelation analysis. It was found that the n_t series was stationary either for $d=0$ and $D=N$, where $N=1, 2, \dots$, or for $d=1$, and $D=1$, or $d=2$ and $D=0$. Furthermore, it was found that the n_t series behaved either as a purely seasonal moving average (MA) process of order $(0, 1, 1)_{12}$ (Eq. (1)) or as an autoregressive moving average (nonseasonal–seasonal) process of order $(0, 1, 1) \times (0, 1, 1)_{12}$ (Eqs. (2) and (3)) or as a nonseasonal autoregressive process of order 2 given as:

$$\begin{aligned} \phi(B)\nabla^d n_t &= a_t \\ \text{where } \phi(B) &= 1 + \phi_{21}B + \phi_{22}B^2, \\ \phi_{21} &= \left(\frac{r_1(1-r_2)}{1-r_1^2} \right), \quad \phi_{22} = \left(\frac{r_2-r_1^2}{1-r_1^2} \right) \end{aligned} \quad (18)$$

and r_i are the autocorrelations at lags $i=1$ and 2. The autoregressive and moving average parameters for the models identified for n_t are given in Table 5.

Figure 5 shows that although a few autocorrelations are outside the confidence limits, the a_t sequences, which are typical of a_t sequences for other models identified, can be considered as random sequences. The values of the ARIMA parameters indicate a persistent pattern with a memory of up to two months with respect to purely non-seasonal effects, and memory of one month for the nonseasonal–seasonal model.

Table 4
Cross correlation weights v_k for some of the bivariate models

Input (d, D)/output (d, D)	v_0	1	2	3	4	5	6	7	8	Standard error: $\pm 2/N$
FSD(0,0)/FSI(2,0)	0.015	0.034	0.143	18.194	-0.065	-0.029	-0.010	0.003	0.033	0.14
$T_{Max}(0,1)/SI(2,0)$	0.265	-0.878	0.394	0.777	-0.779	-1.158	0.097	0.060	-0.361	0.14
$T_{Max}(1,1)/SI(2,0)$	0.072	-0.302	0.015	0.201	-0.042	0.070	0.161	0.060	1.653	0.14
$T_{Min}(0,1)/SI(2,0)$	0.162	-0.452	0.149	-0.279	-0.145	0.126	-0.039	0.033	-1.113	0.14
$T_{Min}(1,1)/SI(2,0)$	-0.284	-0.199	0.058	-0.045	-0.014	0.002	-0.009	-0.065	-0.419	0.14

Table 5
Autoregressive and moving average parameters for the n_t series models

Process: Input(d,D)/output(d,D)	n_t series parameters		ARIMA parameters			Standard error: $\pm 2/\sqrt{N}$
	d	D	ϕ_{21}	ϕ_{22}	θ_{21}	
FSD(0,0)/FSI(2,0)	0	1	–	–	0.40	0.14
FSD(0,0)/FSI(2,0)	1	1	0.01	–	0.11	0.14
TMax(0,1)/SI(2,0)	0	1	–	–	0.30	0.14
TMax(0,1)/SI(2,0)	0	2	–	–	0.25	0.14
TMax(0,1)/SI(2,0)	2	0	–0.70	–0.10	–	0.14
TMax(1,1)/SI(2,0)	0	1	–	–	0.35	0.14
TMax(1,1)/SI(2,0)	0	2	–	–	0.15	0.14
TMax(1,1)/SI(2,0)	2	0	–0.40	–0.05	–	0.14
TMin(0,1)/SI(2,0)	0	1	–	–	0.30	0.14
TMin(0,1)/SI(2,0)	1	1	–0.45	–	–0.10	0.14
TMin(1,1)/SI(2,0)	0	1	–	–	0.40	0.14
TMin(1,1)/SI(2,0)	1	1	–0.55	–	–0.10	0.14

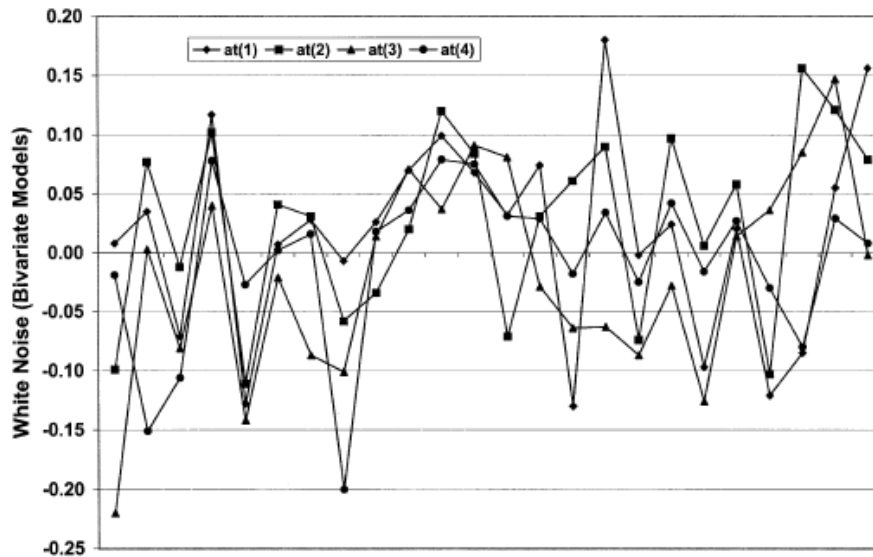


Fig. 5. Autocorrelations of the white noise (residues) for some of the bivariate models. (i) at(1): for n_t (0, 1) from FSD (0, 0)/FSI (2, 0), (ii) at(2): for n_t (0, 2) from T_{Max} (0, 1)/SI (2, 0), (iii) at(3): for n_t (2, 0) from T_{Max} (1, 1)/SI (2, 0), and (iv) at(4): for n_t (1, 1) from T_{Min} (1, 1)/SI (2, 0).

6. Results and discussion

A good understanding of the meteorological phenomena may be achieved through examination of the impulse response weights v_k . The results show that there is a lag of three months between the peak of the differenced fractional sunshine duration and the peak of the differenced fractional solar irradiation, and a lag of at most one month between the peak of differenced maximum temperature and the peak of differenced solar irradiation, but there is no delay between the peak of differenced minimum temperature and the peak of differenced solar irradiation. The noise series follow either a moving average seasonal process of order $(0, 1, 1)_{12}$, or an ARIMA nonseasonal-seasonal process of order $(0, 1, 1) \times (0, 1, 1)_{12}$ or a nonseasonal autoregressive process of order 2. Note that the order of ARIMA models determined for the solar irradiation series (Table 3) agrees with the orders determined from the bivariate models (Table 5). The models identified for the n_t series revealed more information than that obtained in Jain and Lungu [4] in so far as some of the models in this paper exhibit purely nonseasonal behaviour while others exhibit both nonseasonal as well as seasonal behaviour. The relationships between extremum temperatures and solar irradiation are very important for developing countries which lack resources in terms of both equipment and trained manpower. These relationships can be used to generate solar irradiation data, which is required for the sizing of solar devices, from the extremum temperatures. To the best of our knowledge, relationships between extremum temperatures and solar irradiation using Box and Jenkins' [6] techniques are a new contribution, and supplement the commonly used Ångström type relations between sunshine duration and solar irradiation.

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